

# Two-channel Kondo tunneling in triple quantum dot

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The effective spin Hamiltonian of a triple quantum dot with odd electron occupation weakly connected in series with left ( $l$ ) and right ( $r$ ) metal leads is composed of two-channel exchange and co-tunneling terms. Renormalization group equations for the corresponding three exchange constants  $J_l$ ,  $J_r$  and  $J_{lr}$  are solved (to third order). Since  $J_{lr}$  is relevant, the system is mapped on an anisotropic two-channel Kondo problem. The structure of the conductance as function of temperature and gate voltage implies that in the weak and intermediate coupling regimes, two-channel Kondo physics persists at temperatures as low as several  $T_K$ . At even electron occupation, the number of channels equals twice the spin of the triple dot (hence it is a fully screened impurity).

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**Motivation:** In the present work, a simple configuration of localized moment in nanostructures is studied, where the two-channel Kondo Hamiltonian appears in resonance tunneling. Concrete experiment is proposed in order to elucidate the pertinent physics at  $T > T_K$  (the Kondo temperature). In the strong coupling regime, a multichannel Kondo system is known to be a non-Fermi liquid [1], but construction of simple theoretical models pertaining to experimentally feasible setups is notoriously elusive. Examples are magnetic impurity scattering, physics of two-level systems and Kondo lattices (see [2, 3] for review). Recent attempts to realize two-channel Kondo effect in tunneling through quantum dots [4] using peculiar setups still await experimental manifestation. The problem is exemplified in tunneling through a simple quantum dot sandwiched between two metallic "left" ( $l$ ) and "right" ( $r$ ) leads. Starting from the single impurity Anderson model, it is tempting to think of the two leads as a source of two tunneling channels. However, if the two fermion lead operators  $c_{k\sigma a}$  ( $k$  = momentum,  $\sigma$  = spin projection and  $a = l, r$  the lead index) are coupled to a single dot electron operator  $d_\sigma$ , one channel can always be eliminated by an appropriate rotation in  $l - r$  space [5]:

$$\begin{aligned} c_{k\sigma l} &= \cos \theta_k c_{k\sigma 1} + \sin \theta_k c_{k\sigma 2}, \\ c_{k\sigma r} &= -\sin \theta_k c_{k\sigma 1} + \cos \theta_k c_{k\sigma 2}. \end{aligned} \quad (1)$$

As a result, only the standing wave fermions  $c_{k\sigma 1}$  contribute to tunneling. It is therefore natural to expect that a generic two-channel Hamiltonian can be realized only when the rotation (1) cannot eliminate the second channel. Such situation may arise, e.g., when, instead of a simple quantum dot one has a nanoobject consisting of several dots so that  $c_{k\sigma l}$  and  $c_{k\sigma r}$  are coupled to *different* dot electron operators.

**Model Hamiltonian:** Consider a triple quantum dot (TQD) which consists of three dots  $l, f, r$ , connected in series to left and right leads (see Fig. 1). The figure defines also tunneling amplitudes  $V_a$  and hopping amplitudes  $W_a$  between the side dots and the central one. The source of Kondo screening is a Coulomb blockade in the central dot  $f$ , which is supposed to have a smaller radius

(and hence a larger capacitive energy) than the two side dots, namely,  $Q_f \gg Q_{l,r}$  (cf. [6]). The tunneling Hamiltonian has the form  $H_t = \sum_{a=l,r} \sum_{k\sigma} V_a c_{k\sigma a}^\dagger d_{\sigma a} + H.c.$ , and the rotation (1) does not eliminate the second tunneling channel. Note that direct tunneling through the TQD is suppressed due to electron level mismatch and Coulomb blockade, so that *only co-tunneling mechanism* contributes to the current. To quantify these elementary

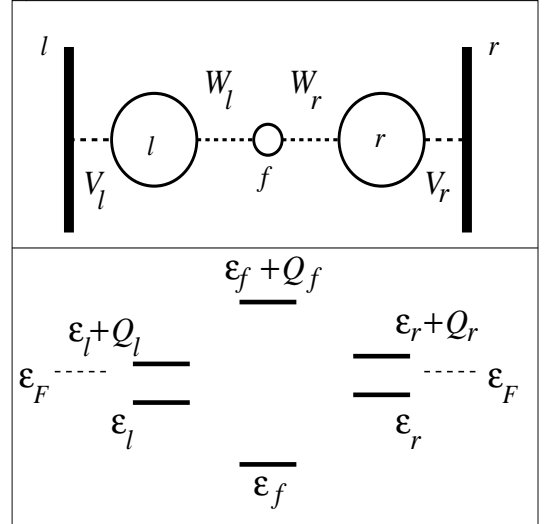


FIG. 1: Triple quantum dot in series (upper panel) and single-electron energy levels of each individual dot  $\epsilon_a = \epsilon_a - V_{ga}$  (bare energy minus gate voltage) (lower panel).

statements consider the case of TQD with three electrons. The Hamiltonian of the isolated TQD reads

$$\begin{aligned} H_d &= \sum_{\alpha=l,r,f} \left( \sum_{\sigma} \epsilon_{\alpha} n_{\sigma\alpha} + Q_{\alpha} n_{\uparrow\alpha} n_{\downarrow\alpha} \right) \\ &+ \sum_{a=l,r} (W_a d_{\sigma a}^\dagger d_{f\sigma} + H.c.), \end{aligned} \quad (2)$$

where  $n_{\sigma a} = d_{\sigma a}^\dagger d_{\sigma a}$ , with  $\sum_{\sigma a} n_{\sigma a} = 3$ . This Hamiltonian can be easily diagonalized in the space of three-electron states  $|\Lambda\rangle$  of the TQD. The lowest ones are classified as a ground state doublet  $|d_1\rangle$ , low-lying doublet

excitation  $|d_2\rangle$  and quartet excitation  $|q\rangle$ . The *single electron* levels  $\varepsilon_a = \epsilon_a - V_{ga}$  are tuned by gate voltages  $V_{ga}$  such that  $\beta_a \equiv W_a/\Delta_a \ll 1$  ( $\Delta_a \equiv \varepsilon_a - \varepsilon_f$ ). To order  $\beta_a^2$ , the corresponding energy levels  $E_\Lambda$  are

$$\begin{aligned} E_{d_1} &= \varepsilon_f + \varepsilon_l + \varepsilon_r - \frac{3}{2} \left[ \frac{W_l^2}{\Delta_l} + \frac{W_r^2}{\Delta_r} \right], \\ E_{d_2} &= E_{d_1} + \left[ \frac{W_l^2}{\Delta_l} + \frac{W_r^2}{\Delta_r} \right], \\ E_q &= \varepsilon_f + \varepsilon_l + \varepsilon_r. \end{aligned} \quad (3)$$

The states  $|d_1\rangle$  and  $|d_2\rangle$  include components with two electrons on  $l$  or  $r$  dots (responsible for co-tunneling). Coupling with the leads admixes three-particle states  $|\Lambda\rangle$  of the dot, with two and four particle states  $|\lambda\rangle$  (for brevity we include only the formers, addition of the latter is straightforward). It is then useful to introduce diagonal and number changing dot Hubbard operators  $X^{\Lambda\Lambda} = |\Lambda\rangle\langle\Lambda|$  and  $X^{\lambda\Lambda} = |\lambda\rangle\langle\Lambda|$  respectively. The Hamiltonian of the whole system (leads, TQD and tunneling) then reads

$$\begin{aligned} H &= \sum_{k\sigma a} \epsilon_{ka} c_{k\sigma a}^\dagger c_{k\sigma a} + \sum_{\Lambda} E_{\Lambda} X^{\Lambda\Lambda} + \sum_{\lambda} E_{\lambda} X^{\lambda\lambda} \\ &+ \left( \sum_{\Lambda\lambda} \sum_{k\sigma a} V_{\sigma a}^{\Lambda\lambda} c_{k\sigma a}^\dagger X^{\lambda\Lambda} + H.c. \right). \end{aligned} \quad (4)$$

Here  $-\frac{D_a}{2} < \epsilon_{ka} < \frac{D_a}{2}$  are lead electron energies with nearly identical bandwidths  $D_l \approx D_r \equiv D_1$ . Within linear response, the Fermi energies are the same on both sides,  $\epsilon_{Fl} = \epsilon_{Fr} = \epsilon_F = 0$ . The tunneling amplitudes  $V_{\sigma a}^{\Lambda\lambda} \equiv V_a \langle \lambda | d_{\sigma a} | \Lambda \rangle$  depend explicitly on the lead index  $a$  and on the respective 2-3 particle quantum numbers  $\lambda, \Lambda$ . Elimination of one of the two channels by the rotation (1) is then impossible: adding  $l$  or  $r$  electron to a given state  $|\lambda\rangle$  results in different states  $|\Lambda\rangle$ .

**RG procedure and spin Hamiltonian:** Following a two-stage RG procedure the low-energy Kondo tunneling through TQD may be exposed. First, the bandwidth is continuously reduced from  $D_1$  to  $D_0 < D_1$  (see [6] and references therein), and the energy levels (3) are renormalized by eliminating high-energy charge excitations [7]. If at the end of this procedure  $E_{d_1}(D_0) < -D_0/2$  and  $E_{d_2}(D_0) - E_{d_1}(D_0)$  exceeds  $T_K$  (defined below), charge fluctuations are quenched and the doublet ground state  $|d_{1\sigma=\uparrow,\downarrow}\rangle$  becomes a localized moment. Following the Schrieffer-Wolff transformation the spin Hamiltonian reads

$$H_s = J_l \mathbf{S} \cdot \mathbf{s}_l + J_r \mathbf{S} \cdot \mathbf{s}_r + J_{lr} \mathbf{S} \cdot (\mathbf{s}_{lr} + \mathbf{s}_{rl}). \quad (5)$$

The spin 1/2 operator  $\mathbf{S}$  acts on  $|d_{1\sigma=\uparrow,\downarrow}\rangle$  whereas the lead electrons spin operators are  $\mathbf{s}_a = \sum_{kk'} c_{k\mu a}^\dagger \boldsymbol{\sigma}_{\mu\nu} c_{k'\nu a}$ . The presence of the left-right spin operator  $\mathbf{s}_{lr} = \sum_{kk'} c_{k\mu l}^\dagger \boldsymbol{\sigma}_{\mu\nu} c_{k'\nu r}$  is responsible for co-tunneling current. Moreover,  $J_a = 8\gamma^2 |V_a|^2 / 3(\epsilon_F - \varepsilon_a) > 0$  (where  $\gamma = \sqrt{1 - \frac{3}{2}(\beta_l^2 + \beta_r^2)}$ ), while  $J_{lr} = -4V_l V_r \beta_l \beta_r / 3(\epsilon_F - \varepsilon_f)$

so that  $|J_{lr}/J_a|$  is of order  $\beta_l \beta_r \ll 1$ . The Hamiltonian (5) then encodes a two-channel Kondo physics, where the leads serve as two independent channels and  $T_K = \max\{T_{Kl}, T_{Kr}\}$ ,  $T_{Ka} = D_0 e^{-D_0/J_a}$ .

In the second stage of the RG procedure, a poor-man scaling technique is used to renormalize the exchange constants by further reducing the band-width  $D_0 \rightarrow D$ . The pertinent fixed points are then identified as  $D \rightarrow T_K$  [8]. Unlike the situation encountered in the single-channel Kondo effect, third order diagrams in addition to the usual single-loop ones should be included (see Fig.5 in Ref. [1] and Fig.9 in Ref. [9]). Below we use  $D_0$  as an energy unit, hence  $J_a, V_a, W_a, V_{ga}, \varepsilon_a, T, D$  etc. now become dimensionless. With  $a = l, r$  and  $\bar{a} = r, l$  the three RG equations for  $J_l, J_r, J_{lr}$  are,

$$\begin{aligned} \frac{dJ_a}{d\ln D} &= -(J_a^2 + J_{lr}^2) + J_a(J_a^2 + J_{\bar{a}}^2 + 2J_{lr}^2), \\ \frac{dJ_{lr}}{d\ln D} &= -J_{lr}(J_l + J_r) + J_{lr}(J_l^2 + J_r^2 + 2J_{lr}^2). \end{aligned} \quad (6)$$

On the symmetry plane  $J_l = J_r \equiv J$ , equations (6) reduce to a couple of RG equations for  $J_{1,2} = J \pm J_{lr}$

$$dJ_i/d\ln D = -J_i^2 + J_i(J_1^2 + J_2^2) \quad (i = 1, 2), \quad (7)$$

subject to  $J_i(D_0 = 1) = J_{i0}$ . These are the well-known equations for the anisotropic two-channel Kondo effect[1]. With  $\phi_i \equiv (J_1 + J_2 - 1)/J_i$ ,  $C_i \equiv \phi_{i0} - \phi_i$  and  $L_i(x) \equiv x - \ln(1 + C_i/x) - 2\ln x$ , the solution of the system (7) is

$$L_i(\phi_i) - L_i(\phi_{i0}) = -\ln D \quad (i = 1, 2). \quad (8)$$

The scaling trajectories in the sector ( $J_l \geq J_r \geq 0, J_{lr} = 0$ ) and in the symmetry plane with  $0 < J_{lr} < J$  are shown in Fig. 2. Although the fixed point  $(1/2, 1/2, 0)$  remains

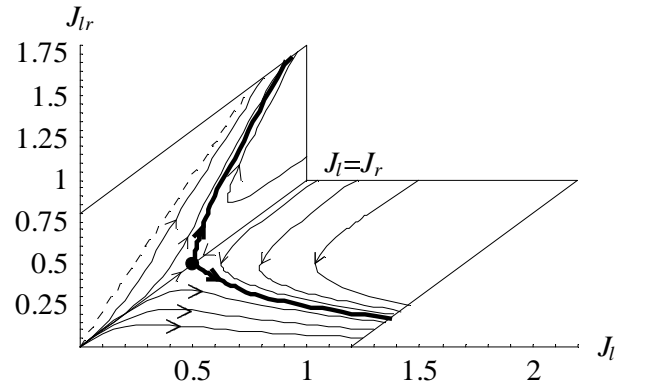


FIG. 2: Scaling trajectories for two-channel Kondo effect in TQD.

inaccessible if  $J_{lr} \neq 0$ , one may approach it close enough starting from an initial condition  $J_{lr0} \ll J_{l0}, J_{r0}$ . Realization of this inequality is a generic property of TQD in series shown in Fig.1.

**Conductance:** According to general perturbative expression for the dot conductance [10], its zero-bias anomaly is encoded in the third order term,

$$G^{(3)} = G_0 J_{lr}^2 [J_l(T) + J_r(T)], \quad (G_0 = \frac{2e^2}{h}). \quad (9)$$

Here the temperature  $T$  replaces the bandwidth  $D$  in the solution (8). Let us present a qualitative discussion of the conductance  $G[J_a(T)]$  (or in an experimentalist friendly form,  $G(V_{ga}, T)$ ) based on the flow diagram 2. (Strictly speaking, the RG method and hence the discussion below, is mostly reliable in the weak coupling regime  $T > T_K$ ). Varying  $T$  implies moving on a curve  $[J_l(T), J_r(T), J_{lr}(T)]$  in three dimensional parameter space (Fig. 2), and the corresponding values of the exchange parameters determine the conductance according to equation (9). Note that if, initially,  $J_{l0} = J_{r0} \equiv J_0$  the point will remain on a curve  $[J(T), J(T), J_{lr}(T)]$  located on the symmetry plane. By varying  $V_{ga}$  it is possible to tune the initial condition  $(J_{l0}, J_{r0})$  from the highly asymmetric case  $J_{l0} \gg J_{r0}$  to the fully symmetric case  $J_{l0} = J_{r0}$ . For a fixed value of  $J_{lr0}$  the conductance shoots up (logarithmically) at a certain temperature  $T^*$  which decreases toward  $T_K$  with  $|J_{l0} - J_{r0}|$  and  $J_{lr0}$ . The closer  $T^*$  is to  $T_K$ , the closer is the behavior of the conductance to that expected in a generic two-channel situation. Thus, although the isotropic two-channel Kondo physics is unachievable in the strong coupling limit, its precursor might show up in the intermediate coupling regime.

The conductance  $G(V_{ga}, T)$  as function of  $T$  for several values of  $V_{ga}$  and the same value of  $J_{lr0}$  is displayed by the family of curves in Fig.3. For  $G$  displayed in curve a,  $T^*/T_K \approx 3$  and for  $T > T^*$  it is very similar to what is expected in an isotropic two-channel system. Alternatively,

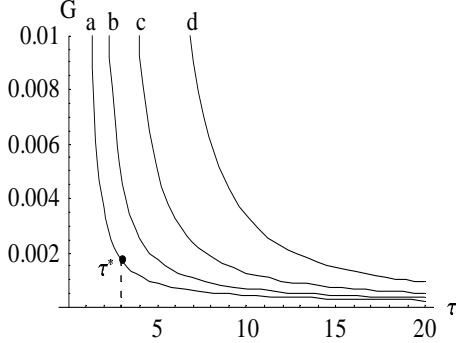


FIG. 3: Conductance  $G$  in units of  $G_0$  as a function of temperature ( $\tau = T/T_K$ ), at various gate voltages. The lines correspond to: (a) the symmetric case  $J_l = J_r$  ( $V_{gl} = V_{gr}$ ), (b-d)  $J_l \gg J_r$ , with  $V_{gl} - V_{gr} = 0.03, 0.06$  and  $0.09$ . At  $\tau \rightarrow \infty$  all lines converge to the bare conductance.

holding  $T$  and changing gate voltages  $V_{ga}$  enables an experimentalist to virtually cross the symmetry plane. This is equivalent to moving vertically downward on Fig. 3. At high temperature the curves almost coalesce and the conductance is virtually flat. At low temperature (still above  $T_K$ ) the conductance exhibits a sharp minimum. This is summarized in Fig.4.

**Higher degeneracy and dynamical symmetries:** The spin Hamiltonian (5) is expressible in terms of the dot (ground-state) spin  $\mathbf{S} = 1/2$  components  $S_i$  ( $i = x, y, z$ ), the generators of the group  $SU(2)$ . Due to com-

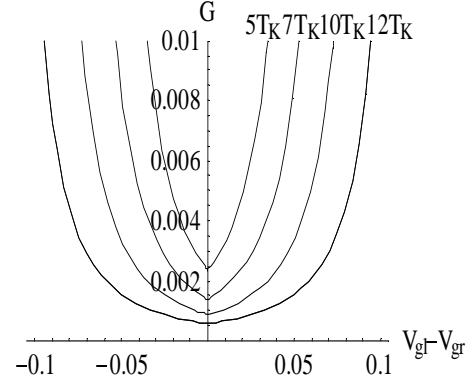


FIG. 4: Conductance  $G$  in units of  $G_0$  as a function of gate voltage at various temperatures (at the origin  $J_l = J_r$ ).

plete spin degeneracy  $E_{d1\uparrow} = E_{d1\downarrow}$ , the dot Hamiltonian itself is of course invariant under  $SU(2)$  but the hybridization leading to the exchange terms  $J_a \mathbf{S} \cdot \mathbf{s}_a$  clearly breaks it, allowing for dot spin-flips. In composite quantum dots, the degeneracy of spin states at the end of the first stage RG procedure might be much richer [6] (and greater than 2). Our analysis shows that for the present model of TQD with  $N = 3$  electrons there exists a scenario of level degeneracy in which the renormalized energies of the two doublets and the quartet are degenerate at the Schrieffer-Wolff limit, that is,  $E_{d1}(D_0) = E_{d2}(D_0) = E_q(D_0) \leq -D_0/2$ . The corresponding wave functions  $|\Lambda\rangle$  are vector sums of states composed of a "passive" electron sitting in the central dot and singlet/triplet (S/T) two-electron states in the  $l, r$  dots. Using certain combinations of dot Hubbard operators  $|\Lambda\rangle\langle\Lambda'|$  one can now define two vector operators  $\mathbf{S}$  and  $\mathbf{M}$  such that  $\mathbf{S}$  is the dot spin 1 operator responsible for transitions within the triplet states, while  $\mathbf{M}$  accounts for S/T transitions[6]. The operators  $\mathbf{S}$  and  $\mathbf{M}$  together with the spin 1/2 operator  $\mathbf{s}$  of the central dot electron generate the group  $SO(4) \times SU(2)$  specified by the Casimir operator  $S^2 + M^2 + s^2 = \frac{15}{4}$ . The corresponding spin Hamiltonian,

$$H_s = \sum_{a=l,r} [J_a^T \mathbf{S} + J_a^{ST} \mathbf{M}] \cdot \mathbf{s}_a + J_{lr} \mathbf{s} \cdot (\mathbf{s}_{lr} + \mathbf{s}_r), \quad (10)$$

expressible in terms of its generators breaks that symmetry. Here  $J_a^T = 4\gamma|V_a|^2/3(\epsilon_F - \epsilon_a) > 0$ ,  $J_a^{ST} = \sqrt{1 - \frac{1}{2}(\beta_l^2 + \beta_r^2)}J_a^T$  and  $J_{lr} = -4V_l V_r \beta_l \beta_r / (\epsilon_F - \epsilon_f)$ , so that  $|J_{lr}|/J_a^T \approx \beta^2$ . Both  $\mathbf{S}$  and  $\mathbf{M}$  vectors are involved in Kondo screening, a situation much richer than the one described by the single impurity Hamiltonian (5). As far as the Kondo physics is concerned, the number of channels  $n = 2$  exactly equals twice the spin,  $2S$ . Therefore, the physics in the strong coupling limit is similar to that of the *single* channel one. The two stable fixed points are  $(J_a^T, J_a^{ST}, J_a^T, J_a^{ST}) = (\infty, \infty, 0, 0)$  implying  $T_K = \max\{T_{Kl}, T_{Kr}\}$ ,  $T_{Ka} = \{\exp[-1/(J_a^T + J_a^{ST})]\}$ . In the weak coupling limit, systems with  $2S > n$  and  $2S = n$  might have different physics.

Finally, let us discuss the experimentally relevant situation of changing electron occupation  $N$  in the same TQD device. For  $N = 2$ , the lowest-energy states consist of two singlets  $|S_a\rangle$  and two triplets  $|T_a\rangle$  (one electron in the left and right dot and one electron in the central dot). The next charged transfer excitons consist of a singlet  $|S_{lr}\rangle$  and a triplet  $|T_{lr}\rangle$  (one electron in the left and right dots). The corresponding energy levels are

$$\begin{aligned} E_{S_a} &= \varepsilon_f + \varepsilon_a - \frac{2W_a^2}{\Delta_a + Q_a} - W_{\bar{a}}\beta_{\bar{a}}, \\ E_{T_a} &= \varepsilon_f + \varepsilon_a - W_{\bar{a}}\beta_{\bar{a}}, \\ E_{S_{lr}} &= E_{T_{lr}} = \varepsilon_l + \varepsilon_r + W_l\beta_l + W_r\beta_r. \end{aligned} \quad (11)$$

where  $\beta_a = W_a/\Delta_a \ll 1$ , ( $\Delta_a = \varepsilon_a - \varepsilon_f$ ).

The most symmetric case is realized when  $\varepsilon_l = \varepsilon_r \equiv \varepsilon$  and  $V_l = V_r \equiv V$ . If at the end of the first stage RG procedure one has  $E_{S_l} \simeq E_{T_l} \simeq E_{S_r} \simeq E_{T_r}$ , the TQD possesses a  $P \times SO(4) \times SO(4)$  symmetry (with  $P$  is  $l \leftrightarrow r$  permutation). Following an RG procedure and a Schrieffer-Wolff transformation, the spin Hamiltonian reads,

$$\begin{aligned} H &= J_1^T \sum_a \mathbf{S}_a \cdot \mathbf{s}_a + J_1^{ST} \sum_a \mathbf{M}_a \cdot \mathbf{s}_a \\ &+ J_2^T \hat{P} \sum_a \mathbf{S}_a \cdot \mathbf{s}_{a\bar{a}} + J_2^{ST} \hat{P} \sum_a \mathbf{M}_a \cdot \mathbf{s}_{a\bar{a}} \\ &+ \sum_{a=l,r} [J_{lr}^T \mathbf{S}_a + J_{lr}^{ST} \mathbf{M}_a] \cdot (\mathbf{s}_{lr} + \mathbf{s}_{rl}), \end{aligned} \quad (12)$$

where  $J_1^T(D_0) = J_2^T(D_0) = \frac{2(1-\beta^2)|V|^2}{\varepsilon_F - \varepsilon}$ ,  $J_{lr}^T(D_0) = \frac{2\beta^2|V|^2}{\varepsilon_F - \varepsilon_f}$ , and  $J^{ST}(D_0) = \sqrt{1 - \frac{2W^2}{(\Delta+Q)^2}} J^T(D_0)$ , and  $\hat{P} = \sum_{a=l,r} (X^{T_a T_{\bar{a}}} + X^{S_a S_{\bar{a}}})$ . Here again, the Kondo physics falls under the category  $2S = n$ . It is then sufficient to write (and solve) second order poor-man scaling equations. Having done it, we find the stable fixed points  $J_i^T, J_i^{ST} \rightarrow \infty$  ( $i = 1, 2$ ). These define the corresponding Kondo temperature

$$T_K = \exp \left( - \frac{2}{(\sqrt{3} + 1)(J_1^T + J_1^{ST})} \right).$$

For  $N = 4$ , similar analysis shows that the TQD again manifests the fully screened Kondo resonance with the  $(\infty, \infty)$  fixed point.

**Summary:** The novel achievements are: i) In composite quantum dots, such as the TQD displayed in Fig. 1, the two-channel (left-right leads) Kondo Hamiltonian (5) emerges in which the impurity is a *real* spin and the current is due solely to co-tunneling. The corresponding exchange constant  $J_{lr}$  is a relevant parameter: by taking even and odd combinations, the system is mapped

on an anisotropic two-channel Kondo problem where  $J_{lr}$  determines the degree of anisotropy. ii) RG equations (6) for the exchange constants are solved (8) and yield the flow diagram displayed in Fig. 2. iii) Although the generic two-channel Kondo fixed-point is not achievable in the strong coupling limit, inspecting the conductance  $G(V_{ga}, T)$  as function of temperature (Fig. 3) and gate voltage (Fig. 4) suggests an experimentally controllable detection of its precursor in the weak and intermediate coupling regimes. iv) Analysis of the Kondo effect in cases of higher spin degeneracy of the dot ground state is carried out in relation with dynamical symmetries. The Kondo physics remains that of a fully screened impurity  $n = 2S$ , and the corresponding Kondo temperatures are calculated. v) Electron occupation is intimately related to the nature of the Kondo physics ( $n > 2S$  for  $N = 3$  and  $n = 2S$  for  $N = 2, 4$ ). vi) One remark regarding non-linear response is in order. If a quantum dot in the two channel Kondo regime is subject to a *finite DC bias* there is a peculiar effect of oscillatory current response [12]. The TQD in series discussed here might then be used to test it experimentally.

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